Spectral Observation in a Forced Mixing Layer

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The dynamics of an incompressible forced mixing layer formed downstream of a splitter plate were investigated. In the regions of zero growth of the mixing layer, spectral analysis of the velocity time fluctuations revealed the existence of locations in which the time variations of the velocity are quasimonochromatic. In these locations the power spectral density distribution presents sideband frequencies associated with the most amplified frequency component, i.e., the forcing frequency, and its harmonics. The sideband frequencies are symmetrically placed with respect to the carrier. The phenomenon also persists with forcing the layer at a frequency equal to either the lower or higher sideband frequency.

Introduction

HOMOGENEOUS mixing layers have been studied in detail, and their dynamics have been found to be controlled by coherent structures[1,6] that originate from the initial two-dimensional Kelvin-Helmholtz instability and also from three-dimensional instability.[7,8] It has been demonstrated that the spatial development of a mixing layer is largely determined by two sets of organized structures: the primary spanwise vortices and the secondary longitudinal counterrotating vortex pairs superimposed upon the spanwise vortices.[9]

In the nonforced condition the typical dimension of these vortices increases linearly with the distance from the splitter plate, resulting in a linear growth of the shear layer. It is believed that both the processes of a "vortex pairing"[3,5] and individual vortex growth play an important role in the shear layer spreading rate. It is also known that the forcing of the most unstable linear mode produces the suppression of the mixing growth and a negative production of turbulent energy.[2,6] In this paper we shall present new measurements of response of the mixing layer to forcing.

Apparatus and Experimental Procedures

The experimental apparatus consisted of a set of small blowers supplying the air, two consecutive settling chambers separated by a first convergence of 12.5:1 contraction ratio, a second convergence of 4:1 contraction ratio, and then a closed test section 25 cm long, 10 cm wide, and 5 cm high. In order to damp the internally generated acoustic noise, the apparatus was built largely of absorbing materials such as plywood and was insulated vibrationally from its supporting structure. The top and bottom walls of the test section were adjusted in order to eliminate the streamwise pressure gradient. The splitter plate, separating the two uniform parallel airstreams, extended upstream through the second convergent into the second settling chamber.

Periodic oscillations were imposed on the layer by the motion of a thin flap at the trailing edge of the splitter plate as shown in Fig. 1. The flap was pivoted at its leading edge and spanned the entire test section. Electromagnets led by a frequency generator drove the flap. The amplitude of the oscillation was maintained constant at 1 mm. To avoid propagation of external noise into the test section and the consequent spurious mixing layer excitation, the test section was enclosed by isolating plexiglass walls.

For the experiments, the velocity ratio U_1/U_2 between the two streams was 0.6 while the average speed $U_a = (U_1 + U_2)/2$ was 3.3 m/s. Using standard hot-wire anemometry techniques, the average velocity field and the average turbulence intensity distributions in the downstream and cross-stream flow direction were measured in order to check the apparatus in absence of forcing. The hot-wire probes were mounted on a y-direction traversing mechanism that could be positioned at six different x locations. The distance between adjacent x stations was kept constant and equal to 2.5 λ_n, where λ_n is the wavelength of the fundamental shear-layer oscillation due to the natural Kelvin-Helmholtz instability.

In Fig. 1 the mean velocity profiles, in the absence of forcing, are shown for different streamwise locations, indicat-

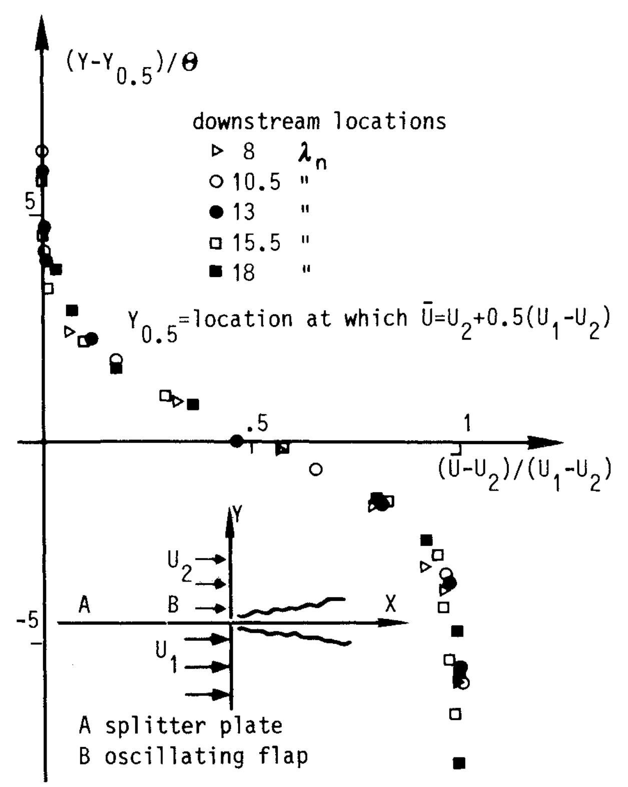

Fig. 1 Velocity profiles for the unperturbed mixing layer and flow schematic.

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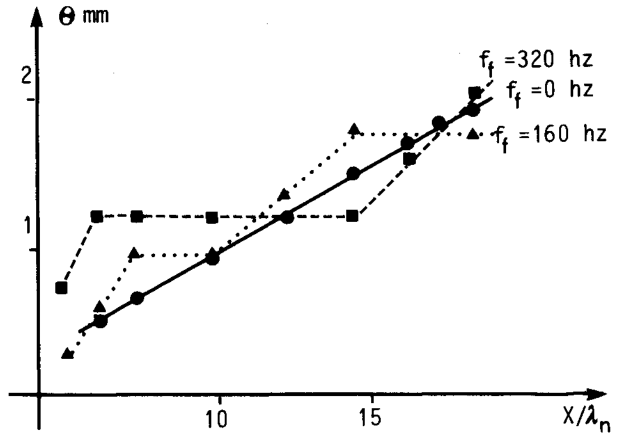

Fig. 2 Variation of the momentum thickness for the unforced and forced mixing layers at $U_2/U_1 = 0.6$.

ing a self-similar flow. The turbulence level at the nozzle exit was measured at 0.5%, and the turbulent intensity distributions agree well with those of other researchers.

Results and Discussion

The frequency f_n of the most amplified unstable oscillation of the velocity field due to the initial Kelvin-Helmholtz instability has been determined experimentally in three different and independent ways. First, the spectrum of the velocity field was measured by a straight hot wire placed immediately downstream of the splitter plate. Second, the initial momentum thickness θ_0 was calculated from the measured initial velocity profile, and the fundamental frequency was deduced from the value of the Strouhal number given by the linear inviscid stability theory ($St = f_n\theta_0/U_a = 0.032$).[3] Third, photographs of smoke flow visualization showed directly the fundamental wavelength $\lambda_n = U_a/f_n$. The fundamental frequency f_n was found to be about 320 Hz with a spread of 1%. To this f_n value corresponds a wavelength λ_n of 10 mm.

The spreading rate of the mixing layer was deduced from the mean velocity measurements, through computation of the variation of the momentum thickness θ along the streamwise direction[2]:

$$\theta = \int_{-\infty}^{\infty} \frac{\bar{U} - U_2}{U_1 - U_2}\left(1 - \frac{\bar{U} - U_2}{U_1 - U_2}\right) \mathrm{d}y$$

In the natural layer θ increased linearly with x, as indicated by the circular points in Fig. 2. The layer was then perturbed at forcing frequencies f_f equal to the fundamental f_n and to its first subharmonic $f_n/2$. A comparison of the growths of the mixing layer for the three different conditions $f_n = 0$ Hz, $f_f = f_n = 320$ Hz, $f_f = f_n/2 = 160$ Hz is shown in Fig. 2. In both forced conditions there is a very rapid initial linear growth due to the strong interaction of the amplified fundamental disturbance with the mean flow as expected from the earlier discovery by Oster and Wygnanski.[2] In the initial part of the layer the subharmonic is always very weak, no matter whether or not the actual forcing frequency is exactly equal to $f_n/2$. There it is the fundamental that picks up most of the energy introduced into the flow. After this initial linear growth the spreading rate is suppressed in both the conditions $f_f = f_n$ and $f_f = f_n/2$. In these plateaus, the fundamental has reached its maximum amplitude. The plateaus are associated with a negative transfer of energy to turbulence; hence, the Reynolds stress distribution measured in these regions is largely negative, as seen in Fig. 3. After these regions the subharmonic amplifies rapidly, extracting energy in turn from the mean flow, which resumes growth of the shear layer. In the case of subharmonic forcing the layer reaches a second plateau, which indicates that the subharmonic too has reached its maximum amplitude. This situation is typical of mixing layers perturbed at a frequency equal to the subharmonic of the fundamental, as also was found experimentally by Ho and Huang[11] and analytically by Nikitopoulos and Liu.[10]

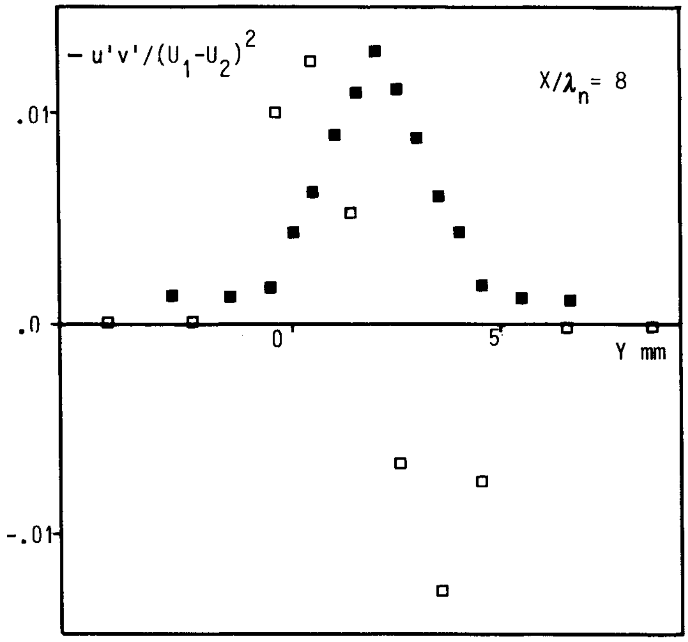

Fig. 3 Reynolds stress cross-stream distributions: ■ $f_f = 0$ Hz, □ $f_f = 160$ Hz.

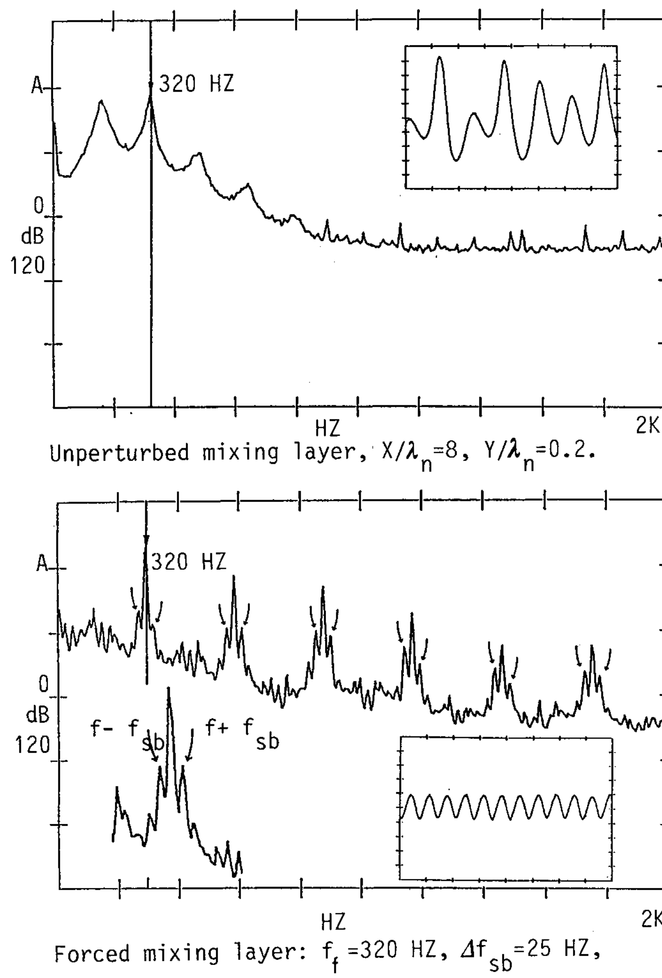

Fig. 4 Time spectra of the velocity signal. In the box inside the graphs, the time variation of the signal is shown: a) $f_f = 0$ Hz, $X/\lambda_n = 8$, $Y/\lambda_n = 0.2$; b) $f_f = 320$ Hz, $\Delta f_{sb} = 25$ Hz, $X/\lambda_n = 8$, $Y/\lambda_n = 0.2$.

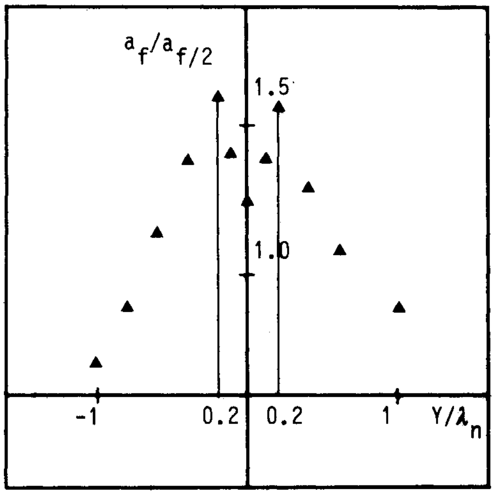

Fig. 5 Cross-stream distribution of the spectral amplitude ratio between the components at the forcing frequency and its first subharmonic $X/\lambda_n = 8$.

In the regions in which it is possible to suppress the growth of the layer, our measurements revealed that there are certain locations inside the layer in which the velocity signal has an impressive coherent time variation (see Fig. 4b). These regions are the parts of the layer where the fundamental saturates, resonating with the forcing frequency $f_f = f_n$, and also further downstream where the subharmonic saturates, resonating with the forcing frequency $f_f = f_n/2$. The velocity is quasisinusoidal in time, with a slight modulation of amplitude. The corresponding spectrum shows, symmetrically displaced with respect to the most amplified spectral component and its harmonics, two peaks of lower amplitude. This "sideband frequency interval" Δf_{sb} that separates these sideband peaks from their associated main spectral component ranged typically from 25 to 40 Hz, or about 10% of the fundamental component. The presence of the sidebands represents an energy transfer from the resonance frequency, the fundamental in this case, to a close resonance frequency. The interaction of $f_f + \Delta f_{sb}$ with f_n also produces the frequency $f_n - \Delta f_{sb}$, and vice versa. The same fact holds with strong regularity at the harmonic frequencies.

This phenomenon was evident in the region where the fundamental saturates. When forcing at the subharmonic frequency, in the region where the subharmonic saturates, the sidebands are still noticeable but not as pronounced as in the previous case. This is perhaps because of partial loss of the spatial coherence due to the occurrence of the small-scale three-dimensional transition taking place upstream of the region of subharmonic resonance, corresponding to the second vortex merging interaction. The phenomenon persists when forcing the layer at a frequency equal to one of the sideband frequencies, $f_f = f_n + \Delta f_{sb}$ or $f_f = f_n - \Delta f_{sb}$. The spectrum again displays sideband peaks on either side of the forcing frequency and its harmonics.

The phenomenon was observed close to the centerline of the layer. Moving the sensor in the y direction, we located these points as 1/5 λ_n apart from the centerline: actually corresponding to the two peaks of the cross-stream eigenfunction, giving the amplitude ratio between the forcing frequency and its subharmonic (see Fig. 5). This displacement remains constant because we are in the regions of suppressed flow growth, where there is no information transported laterally.

In Fig. 4a the corresponding situation in the unperturbed layer is shown at exactly the same streamwise and cross-stream position. The sidebands are not apparent in the spectrum, and the time velocity fluctuations, though not completely random, are not as regular as in the forced case.

The source of this phenomenon is evidently the inherent nonlinear coupling among the various oscillation modes present in such a flow. Very recently a similar phenomenon, in the case of subharmonic forcing, was discovered analytically by Monkewitz.[12] He studied the nonlinear interaction between the fundamental mode and its first subharmonic in an inviscid parallel shear layer using a quasilinear stability analysis. When the fundamental is taken exactly neutral, he shows that in exciting a close sideband of the subharmonic, the other symmetric sideband is also emerging from the solution of the equation describing the spatial-temporal subharmonic amplitude evolution.

Conclusions

Using spectral analysis, we observed the response of the forced mixing layer in both the regions of linear growth and no growth. The experiments revealed that in the resonance regions, where first the fundamental frequency saturates and then in turn the subharmonic saturates, the velocity signal was highly periodic in certain locations and showed a spectrum characterized by the presence of distinct sideband frequencies. This phenomenon can be described as a spectral energy transfer from the local resonance frequency to the two sidebands.

Theoretical support for these experimental observations has recently come from Monkewitz[12] in a paper dealing with the nonlinear mode interaction in inviscid parallel mixing layers. For the case of subharmonic resonance, he showed an analog one situation of sideband excitation.

Phenomenologically, these findings also could be associated with the presence of secondary three-dimensional streamwise vortical structures formed by counterrotating vortex pairs, which propagate under the combined effects of induction and interaction with the continuous shear and the main spanwise vortex system.

Acknowledgments

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